239. The Condensed Phosphoric Acids and Their Salts. Part VII.* Ion Association between Some Tervalent Cations and the Tri- and Tetra-metaphosphates.

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The extent of ion-pair formation between lanthanum, some complex cobaltic cations, and the trimetaphosphate and tetrametaphosphate ions has been calculated from the conductivities of mixed solutions of the cation chlorides containing solutions of the sodium metaphosphates. The resulting dissociation constants are qualitatively related to the cation sizes.

In continuation of previous studies of the extent of ion-pair formation between cations and $\mathrm{P}_{3} \mathrm{O}_{9}{ }^{3-}$ and $\mathrm{P}_{4} \mathrm{O}_{12}{ }^{4-}$ ions (see Part VI ${ }^{*}$ ), attention has now been directed to the kind of interaction that occurs when tervalent cations are used. The same procedure has been adopted as in the previous cases, namely use of the differences between the calculated and observed specific conductivities of mixtures of the sodium metaphosphates and the cation chlorides.

Considering, first, the trimetaphosphates, where $\mathrm{MP}_{3} \mathrm{O}_{9}$ ion-pairs are formed ( M representing a tervalent cation), for a solution containing $c_{1}$ equiv. per 1 . of sodium trimetaphosphate and $c_{2}$ of $\mathrm{MCl}_{3}$, if $a$ g.-ions of $\mathrm{NaP}_{3} \mathrm{O}_{9}{ }^{2-}, b \mathrm{~g}$.-ions of $\mathrm{MCl}^{2+}$, and $d \mathrm{~g}$.-ions of $\mathrm{MP}_{3} \mathrm{O}_{9}$ are present per l., then

$$
\begin{aligned}
&\left.10^{3} \kappa \text { (obs. }\right)=\Lambda_{\mathrm{Na}+}\left(c_{1}-a\right)+3 \Lambda_{\mathrm{P}_{3} \mathrm{O}^{2}-}\left(c_{1} / 3-a-d\right) \\
&+2 a \Lambda_{\mathrm{NaP}_{3} \mathrm{O}^{2}-}+3 \Lambda_{\mathrm{M}^{+}+}\left(c_{2} / 3-b-d\right)+\Lambda_{\mathrm{Cl}^{-}}-\left(c_{2}-d\right)+2 b \Lambda_{\mathrm{MCl}^{2}+}
\end{aligned}
$$

where $\kappa$ is the specific conductivity. The value of $a$ is slightly different from what it would be in sodium trimetaphosphate solutions of the same ionic strength as the mixtures, but for the purposes of calculation this difference may be neglected, and similarly for $b$ (both $a$ and $b$ are relatively small). Accordingly, the above expression becomes

$$
10^{3} \kappa \text { (obs.) }=c_{1} \Lambda_{\mathrm{Na}_{3} \mathrm{P}_{3} \mathrm{O},}+c_{2} \Lambda_{\mathrm{MCl}_{2}}-3 d\left(\Lambda_{\mathrm{P}_{3} \mathrm{O}_{2}-}+\Lambda_{\mathrm{M}^{2}+}\right)
$$

where the first two equivalent conductivities $(\Lambda)$ can be found from known data, and those of the two ions from the appropriate Onsager equation $\Lambda_{i}=\Lambda_{i}{ }^{\circ}-B I^{\frac{1}{2}}, I$ being the ionic

* Part VI, preceding paper.
strength and $B$ the theoretical Onsager value. The values of $\mathrm{A}_{6}{ }^{\circ}$, the equivalent ionic conductivity at zero ionic strength, and of $B$ are listed in Table 2. The ionic strength is given by

$$
I=\delta \cdot 5\left\{4 c_{1}+4 c_{2}-6 a-5 d-18 d\right\}
$$

$a$ being obtained from the known dissociation constant of $\mathrm{NaP}_{3} \mathrm{O}_{9}{ }^{2-}$ (Davies and Monk, $J ., 1949,413$ ) and $b$ from the corresponding data for $\mathrm{MCl}^{2+}$ in the cases of the complex cobaltic chlorides (Jenkins and Monk, $J$., 1951, 68). The other tervalent chloride used, lanthanum chloride, appears to ionise fully in the concentration range used here (James and Monk, Trans. Faraday Soc., 1950, 46, 141). The true values of $a, b$, and $d$ were found by successive approximations.

If no $\mathrm{MP}_{3} \mathrm{O}_{9}$ were formed, we would have

$$
10^{3} \kappa(\text { calc. })=c_{1} \Lambda_{\mathrm{Na}_{3} \mathrm{P}_{3} \mathrm{O}_{9}}+c_{2} \Lambda_{\mathrm{MCl}_{2}}
$$

Hence, by subtraction,

$$
\left.10^{3}\{\kappa \text { (calc.) }-\kappa \text { (obs. })\right\}=3 b\left(\Lambda_{\mathrm{P}_{3}, \mathrm{O}^{3}-}+\Lambda_{\mathrm{M}^{1}}\right)
$$

which gives $b$. The final values of $a, b$, and $d$ thus obtained were used in calculating the dissociation constants of the equilibria
i.e.,

$$
\mathrm{MP}_{3} \mathrm{O}_{9} \rightleftharpoons \mathrm{M}^{3+}+\mathrm{P}_{3} \mathrm{O}_{9}{ }^{3-},
$$

$$
K=\left(c_{2} / 3-b-d\right)\left(c_{1} / 3-a-d\right) f_{1} f_{2} / b
$$

where the ion activity coefficients $f_{1}$ and $f_{2}$ are found from the Debye-Hückel limiting equation $-\log f_{i}=0.509 z_{i} I^{\ddagger}, z_{i}$ being the ion valency. The calculated specific conductivities were obtained from known data (Davies and Monk; Jenkins and Monk; James and Monk, locc. cit.) for sodium trimetaphosphate, cobaltammine chlorides, and lanthanum chloride respectively. The average dissociation constants from Table $\mathbf{l}$ (which contains the experimental results and the calculated data) are given in Table 4.

Table 1. Data for the dissociation constants of $\mathrm{MP}_{3} \mathrm{O}_{9}$.

| $10^{4} C_{1}$ | $10^{4} c_{2}$ | $10^{8} a$ | $10^{5} b$ | $10^{\text {® }}$ d | $10^{4} \kappa$ (calc.) | $10^{4} \kappa$ (obs.) | $10^{3} I$ | $10^{5} \mathrm{~K}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{M}=$ Lanthanum |  |  |  |  |  |  |  |  |
| 17-640 | 17.465 | 1.0 | 53.23 | - | $4 \cdot 5645$ | $2 \cdot 3152$ | 2.22 | $0 \cdot 19$ |
| $9 \cdot 614$ | 6.303 | $2 \cdot 0$ | 20.29 | -- | $2 \cdot 0851$ | $1 \cdot 2103$ | $1 \cdot 35$ | $0 \cdot 19$ |
| 17.938 | 11.483 | $5 \cdot 0$ | 37-27 | - | $3 \cdot 7965$ | $2 \cdot 2006$ | $2 \cdot 51$ | $0 \cdot 21$ |
| 17.798 | 14.292 | $2 \cdot 5$ | 45.78 | - | 4-1492 | $2 \cdot 2157$ | $2 \cdot 29$ | $0 \cdot 20$ |
| $\mathrm{M}=$ Hexamminocobalt(III) |  |  |  |  |  |  |  |  |
| 11.709 | 6.926 | $2 \cdot 9$ | 15.93 | $1 \cdot 1$ | $2 \cdot 5768$ | $1 \cdot 7668$ | 2.28 | $3 \cdot 69$ |
| 11.498 | 14.786 | 1.5 | 25.41 | $7 \cdot 4$ | $3 \cdot 8403$ | 2.5101 | 2.95 | $3 \cdot 70$ |
| $10 \cdot 028$ | $4 \cdot 113$ | 2.6 | $9 \cdot 84$ | $0 \cdot 4$ | 1.9263 | $1 \cdot 4231$ | 1.94 | $3 \cdot 58$ |
| 9.999 | $6 \cdot 332$ | $2 \cdot 1$ | 14.09 | 1.0 | $2 \cdot 2844$ | $1 \cdot 5646$ | 2.00 | $3 \cdot 64$ |
| $\mathrm{M}=$ Trisethylenediaminocobalt(III) |  |  |  |  |  |  |  |  |
| 4.062 | $2 \cdot 909$ | $0 \cdot 4$ | 5.01 | 1.0 | 0.9334 | 0.7076 | 0.94 | 4.07 |
| 4.023 | $5 \cdot 159$ | $0 \cdot 3$ | $7 \cdot 37$ | 1.2 | $1 \cdot 2435$ | 0.9134 | $1 \cdot 16$ | 3.90 |
| $3 \cdot 247$ | 1.228 | $0 \cdot 3$ | 2.27 | 0.7 | 0.5934 | $0 \cdot 4904$ | 0.67 | 3.80 |
| $3 \cdot 229$ | $2 \cdot 495$ | $0 \cdot 3$ | $3 \cdot 97$ | 0.8 | $0 \cdot 7718$ | 0.5918 | 0.78 | 3.99 |
| $\mathbf{M}=$ Trispropylenediaminocobalt(III) |  |  |  |  |  |  |  |  |
| 9.768 | $7 \cdot 360$ | 2.5 | 6.60 | $3 \cdot 6$ | $2 \cdot 1464$ | 1.8788 | $2 \cdot 81$ | $24 \cdot 4$ |
| $9 \cdot 690$ | $15 \cdot 201$ | $2 \cdot 0$ | 10.22 | $15 \cdot 0$ | 3.0893 | $2 \cdot 6837$ | 4.06 | 21.7 |
| 11.705 | $5 \cdot 524$ | $3 \cdot 5$ | $5 \cdot 58$ | 1.9 | $2 \cdot 0269$ | $1 \cdot 8007$ | 2.73 | $22 \cdot 4$ |
| 10.583 | 16.918 | $2 \cdot 3$ | 11.48 | 18.0 | $2 \cdot 9411$ | 2-3949 | $4 \cdot 46$ | 21.7 |

Table 2. Constants used in the calculations.

| $\Lambda_{i}{ }^{\circ}$ | $\begin{aligned} & \mathrm{La}^{3+} \\ & \mathbf{6 9 . 6 1} \end{aligned}$ | $\begin{gathered} \mathrm{Co}\left(\mathrm{NH}_{3}\right)_{8}{ }^{\mathbf{3 +}} \\ \mathbf{9 9 . 2 0} \end{gathered}$ | $\begin{gathered} \mathrm{Co}(\mathrm{en})_{3}{ }^{\mathrm{sir}-} \\ \mathbf{7 4 \cdot 7 1} \end{gathered}$ | $\begin{gathered} \mathrm{Co}(\mathrm{pn})_{3}{ }^{3+} \\ \mathbf{6 5 . 0 6} \end{gathered}$ | $\begin{array}{r} \mathrm{P}_{3} \mathrm{O}_{9}{ }^{3-} \\ 83 \cdot 59 \end{array}$ | $\begin{gathered} \mathrm{P}_{4} \mathrm{O}_{12}{ }^{4-} \\ 93.69 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $B$ | $127 \cdot 6$ | 147.5 | 130.9 | 124.7 | 141.0 | 190.8 |
|  | $\mathrm{LaCl}_{3}$ | $\mathrm{Co}\left(\mathrm{NH}_{3}\right)_{6} \mathrm{Cl}_{3}$ | $\mathrm{Co}(\mathrm{en})_{3} \mathrm{Cl}_{3}$ | $\mathrm{Co}(\mathrm{pn})_{3} \mathrm{Cl}_{3}$ | $\mathrm{Na}_{3} \mathrm{P}_{3} \mathrm{O}_{9}$ | $\mathrm{Na}_{4} \mathrm{P}_{4} \mathrm{O}_{12}$ |
| $\Lambda^{\circ}$ | $145 \cdot 95$ | $175 \cdot 55$ | $151 \cdot 05$ | $141 \cdot 40$ | $133 \cdot 70$ | $143 \cdot 80$ |
| E | 198.6 | $260 \cdot 0$ | $254 \cdot 0$ | 232.0 | 212.0 | $313 \cdot 0$ |

( $E$ represents the experimental slope in the equation $\Lambda=\Lambda^{\circ}-E I^{\ddagger}$; for $B$, see p. 1317.)

The method of calculating the concentrations of $\mathrm{MP}_{4} \mathrm{O}_{12}{ }^{-}$ion-pairs in mixtures of $\mathrm{MCl}_{3}$ and $\mathrm{Na}_{4} \mathrm{P}_{4} \mathrm{O}_{12}$ is similar to that described above, except that the type of assumption used previously (Part V, loc. cit.) is used, namely, that the equivalent conductivity of the $\mathrm{MP}_{4} \mathrm{O}_{12}{ }^{-}$ion is one-quarter of that of the $\mathrm{P}_{4} \mathrm{O}_{12}{ }^{4-}$ ion. The ionic-strength expression is

$$
I=0.5\left\{5 c_{1}+4 c_{2}-8 a-9 e-24 b\right\}
$$

where $c_{1}$ now represents the equivalent concentration of sodium tetrametaphosphate $c_{2}$ is that of $\mathrm{MCl}_{3}, a$ and $b$ are the $\mathrm{NaP}_{4} \mathrm{O}_{12}{ }^{3-}$ and $\mathrm{MP}_{4} \mathrm{O}_{12}{ }^{-}$concentrations respectively. The latter is obtained from

$$
\left.10^{3}\{\kappa \text { (calc.) }-\kappa \text { (obs. })\right\}=b\left(3.75 \Lambda_{\mathrm{P}_{4} \mathrm{o}_{\mathbf{1}} \mathbf{a}^{4}-}+3 \Lambda_{\mathrm{M}^{2}+}\right)
$$

The dissociation constants $K$ were calculated from

$$
K=\left(c_{2} / 3-b-d\right)\left(c_{1} / 4-a-b\right) f_{1} f_{2} / b f_{3}
$$

where the activity coefficients $f_{i}$ were calculated from the expression given previously. The data are given in Table 3.

Table 3. Data for the dissociation constants of $\mathrm{MP}_{4} \mathrm{O}_{12}{ }^{-}$.


## Discussion

A summary of the dissociation constants is given in Table 4, together with those of the corresponding sulphates (Jenkins and Monk, loc. cit.). In each series it will be noticed that

Table 4.

|  | $\mathrm{La}^{\text {s+ }}$ | $\mathrm{Co}\left(\mathrm{NH}_{3}\right)^{3+}$ | $\mathrm{Co}(\mathrm{en})_{3}{ }^{\text {+ }}$ | $\mathrm{Co}(\mathrm{pn})_{3}{ }^{\text {+ }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Average dissociation constants |  |  |  |  |
| Sulphates ( $\times 10^{\mathbf{4}}$ ) | $2 \cdot 0$ | 2.77 | 3.55 | 17.5 |
| Trimetaphosphates ( $\times 10^{5}$ ) | $0 \cdot 20$ | $3 \cdot 65$ | $3 \cdot 94$ | $22 \cdot 6$ |
| Tetrametaphosphates ( $\times 10^{6}$ ) | $0 \cdot 22$ | 1.80 | $1 \cdot 69$ | $25 \cdot 6$ |
| Bjerrum distances ( $\AA$ ) |  |  |  |  |
| Sulphates | 3.8 | $4 \cdot 0$ | $4 \cdot 2$ | $7 \cdot 3$ |
| Trimetaphosphates | $3 \cdot 9$ | $5 \cdot 3$ | $5 \cdot 3$ | 7.7 |
| Tetrametaphosphates | $4 \cdot 7$ | $5 \cdot 5$ | $5 \cdot 5$ | $7 \cdot 6$ |
| Radii of ion-pairs from Stokes's law ( $\AA$ ) |  |  |  |  |
| Sulphates | 6.2 | $5 \cdot 1$ | 6.0 | 6.5 |
| Trimetaphosphates | $7 \cdot 2$ | $6 \cdot 1$ | $7 \cdot 0$ | $7 \cdot 5$ |
| Tetrametaphosphates ......... | $7 \cdot 9$ | 6.7 | $7 \cdot 6$ | 8.1 |

the values for lanthanum are the smallest, those for $\mathrm{Co}\left(\mathrm{NH}_{3}\right)_{6}{ }^{3+}$ and $\mathrm{Co}(\mathrm{en})_{3}{ }^{3+}$ show little difference, and those for $\mathrm{Co}(\mathrm{pn})_{3}{ }^{\mathbf{3 +}}$ are much the largest. The Bjerrum distances of closest approach of the ions, and the radii of the ion-pairs as calculated by Stokes's law from the limiting mobilities of the ions, are also included in Table 4.

The difference between the radii of the ion-pairs as calculated by the two methods is most marked with the lanthanum series, and supports the suggestion made previously (Part IV, loc. cit.; also Part VI), namely, that with these high valency forces the energy of interaction is sufficiently large to overcome, at least, in part, the energy of ion-hydration. The lanthanum ion is undoubtedly heavily hydrated, since the crystallographic radius is $1-15 \AA$ (Pauling, " Nature of the Chemical Bond," Cornell Univ. Press, 2nd edn., p. 346) whereas application of Stokes's law gives the ion radius as $3.95 \AA$. Using the $1 \cdot 15 \AA$ value together with the Stokes's law values for the three anions, we get $3 \cdot 4,4 \cdot 4$, and $5 \cdot 1 \AA$ for the radii of the lanthanum ion-pairs, in much better agreement with the Bjerrum distances. This would only be admissible if the anions were hydrated to a relatively small extent, and this conclusion is upheld by the trimetaphosphate results (Part VI, loc. cit.). Some reserve must be placed on this conclusion, however, since the comparison with the tetrametaphosphates of bivalent metals does not fit into this scheme (probably because of the complication that two equilibria are involved).

The relatively close agreement between the Bjerrum and the Stokes's law distances found with the complex cobaltic ions indicates that both cations and anions are comparatively unhydrated. This applies particularly to the last of these cations, $\mathrm{Co}(\mathrm{pn})_{3}{ }^{3+}$, where, by analogy with the alkali and alkaline-earth metals, we would expect the largest ion of the series to be the least solvated.

## Experimental

The conductivity equipment used was that referred to in Part VI, where the sources of the sodium metaphosphates were also given. The lanthanum chloride was made by boiling an excess of " Spec-pure" lanthanum oxide with a $50 \%$ solution of A.R. hydrochloric acid until the pH was about $5 \cdot 5$, filtering it, and diluting it with conductivity water. The complex cobaltic salts were taken from samples used for previous conductivity work (Jenkins and Monk, loc. cit.). In order to obtain the concentrations of the stock solutions of the salts, conductivity measurements were made with each one, and the concentrations found by interpolation from the known data.

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